**MATHEMATICS METHODS**

**MAWA Semester 1 (Unit 3) Examination 2020**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **June 12th the end of week 7 of term 2, 2020**

**Section One: Calculator-assumed (100 Marks)**

**Question 8(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses initial condition to construct equation and solves for *A* * constructs equation related to half life * solves for | 1  1  1 |

**Question 8(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| For isotope B, .    Solving    Hence approximately 137 years from now. | |
| Mathematical behaviours | Marks |
| * states equation to be solved * solves for *t* and states time (in years) | 1  1 |

**Question 9(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| as required. | |
| Mathematical behaviours | Marks |
| * anti-differentiates the derivative function correctly * uses the initial condition to find the constant of integration and deduces the required solution | 1  1 |

**Question 9(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)  As  i.e. population of infected people increases indefinitely  (ii)  As  i.e. population of infected people stabilises to 25  (iii) | |
| Mathematical behaviours | Marks |
| (i)   * recognises that when *K* = 1, and as   ie population of infected people increases indefinitely  (ii)   * recognises that when *K* = 0 and as   ie population of infected people stabilises to 25  (iii)   * correct graph for *K*= 0 * correct graph for *K* = 1 | 1  1  1  1 |

**Question 9(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence,  From CAS, this is a decreasing function.    ie Population of infected people will reduce to zero after 2.0345 weeks | |
| Mathematical behaviours | Marks |
| * uses *t* = 1 and *P* = 25 to find the correct value of *K* * uses *P* = 0 to find the value of *t* * states a valid prediction | 1  1  1 |

**Question 10(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Area of triangle  Hence, | |
| Mathematical behaviours | Marks |
| * determines area of triangle as an exact value * states formula for total area in terms of *x* * clearly demonstrates rearrangement of formulae to achieve required   result. | 1  1  1 |

**Question 10(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence the maximum area is approximately 5.86 m2. | |
| Mathematical behaviours | Marks |
| * equates to 0 and solves * determines or otherwise justifies maximum * calculates maximum area | 1  1  1 |

**Question 11(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * equates the sum of probabilities to 1 * evaluates *b* * states expression for * evaluates *a* | 1  1  1  1 |

**Question 11(b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)    (ii)  Standard deviation of standard deviation of | |
| Mathematical behaviours | Marks |
| (i)   * states expression to determine the variance of *X* * evaluates variance * evaluates standard deviation   (ii)   * states correct result | 1  1  1  1 |

**Question 12(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  | | --- | --- | --- | |  |  |  | |  |  |  | |  |  |  | |  |  |  | |  |  |  | | |
| Mathematical behaviours | Marks |
| * completes two rows of the table correctly * completes all rows of the table correctly | 1  1 |

**Question 12(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| correct to decimal places. | |
| Mathematical behaviours | Marks |
| * evaluates limit correctly | 1 |

**Question 12(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)    (ii) | |
| Mathematical behaviours | Marks |
| (i)   * states solution   (ii)   * states exact solution | 1  1 |

**Question 13(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | *x* | 1 | 2 | 3 | 4 | | *y* | 2 |  |  |  | | |
| Mathematical | Marks |
| * states all three correct values | 1 |

**Question 13(b)**  **(4 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)  Area of lower rectangles    (ii)  Area of upper rectangles | |
| Mathematical behaviours | Marks |
| (i)   * sums the correct rectangles * deduces correct result   (ii)   * sums the correct rectangles * evaluates correctly | 1  1  1  1 |

**Question 13(c)**  **(1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Estimated area | |
| Mathematical behaviours | Marks |
| * calculates the average of the lower and upper areas | 1 |

|  |  |
| --- | --- |
| Solution | |
| Area under the curve is | |
| Mathematical behaviours | Marks |
| * states the correct answer | 1 |

**Question 13(d) (1 mark)**

**Question 14(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Some people would read both digital and print. If these entries are summed those people will be counted twice. | |
| Mathematical behaviours | Marks |
| * recognises that some people will read both forms of publication | 1 |

**Question 14(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses correct numerator * uses correct denominator and deduces result | 1  1 |

**Question 14(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states Binomial * calculates mean * calculates variance | 1  1  1 |

**Question 14(d) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)    (ii)    (iii) | |
| Mathematical behaviours | Marks |
| (i)   * states correct probability   (ii)   * states appropriate probability expression * calculates probability   (iii)   * states correct expression for first three outcomes * states fourth outcome and calculates probability | 1  1  1  1  1 |

**Question 14(e) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let be the random variable denoting the number of people in the 200 who read print media. | |
| Mathematical behaviours | Marks |
| * changes parameter of distribution * states appropriate probability statement * evaluates | 1  1  1 |

**Question 15(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * applies linearity for derivatives * applies the Fundamental Theorem and evaluates, stating the correct result | 1  1 |

**Question 15(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * differentiates both sides of the equation (or applies result from part (a)) * determines result | 1  1 |

**Question 16(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 16(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Distance travelled in the first 6 seconds is 50 metres | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 16(c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Displacement after 12 seconds is 65 m | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 16(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Distance travelled after 12 seconds is 10+40+20+5 =75 m | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 16(e) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| At  both the velocity and the acceleration are negative hence the particle is speeding up. | |
| Mathematical behaviours | Marks |
| * states the particle is speeding up | 1 |

**Question 17(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  | | --- | --- | --- | | *y* | 0 | 1 | | P(*Y=y*) |  |  | | |
| Mathematical behaviours | Mark |
| * completes first probability correctly * completes second probability correctly | 1  1 |

**Question 17(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| It is a Bernoulli distribution with mean = | |
| Mathematical behaviours | Marks |
| * states the distribution name * states the mean | 1  1 |

**Question 17(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| or *2* | |
| Mathematical behaviours | Marks |
| * states all values | 1 |

**Question 17(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| (not prime and not prime) | |
| Mathematical behaviours | Mark |
| * calculates probability | 1 |

**Question 17(e) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence *X*=1 is the most likely result. | |
| Mathematical behaviours | Mark |
| * calculates * calculates * states correct conclusion | 1  1  1 |

**Question 17(f) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let the random variable represent the operator’s financial position for each game.   |  |  |  | | --- | --- | --- | | *f* | -2 | 5 | |  |  |  |     Hence the operator will expect to make a profit of $1.36 per game in the long term. With 500 contestants he will expect to make | |
| Mathematical behaviours | Marks |
| * determines expected value for 1 game * calculates gain for the day * states final outcome, with unit and explains | 1  1  1 |

**Question 17(g) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let be the charge to play the game.   |  |  |  | | --- | --- | --- | | *f* | (*k*-7) | *k* | |  |  |  |     Hence the operator would need to charge $3.64. | |
| Mathematical behaviours | Marks |
| * constructs equation for expected value * solves equation to determine | 1  1 |

**Question 18(a) (6 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)  Since the maximum and minimum values are 14.5 and 9.5  and and  or  mean line and amplitude  Since the period of the oscillation is 12,  (ii)    Maximum at *t* = 11.7 | |
| Mathematical behaviours | Marks |
| (i)   * explains exactly one of *a* and *b* values * explains both *a* and *b* values * identifies the period to explain the value of *c*   (ii)   * differentiates correctly * equates to *0* and equates angle to * solves equation to determine *d* | 1  1  1  1  1  1 |

**Question 18(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| On April 30th,  hours  So we can expect hours of sunlight on April 30th. | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 18(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| So average =  So the average daily amount in May and June is hours | |
| Mathematical behaviours | Marks |
| * uses correct integral * states solution | 1  1 |

**Question 18(d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence the maximum value of is  hours per month  Using the increments formula , the maximum change in successive days is approximately hours.  i.e. 2.62 minutes (or minutes to the nearest minute). | |
| Mathematical behaviours | Marks |
| * identifies maximum value of * substitutes into increments formula correctly * states answer to the nearest minute | 1  1  1 |

**Question 19(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| represents the area bounded by the curve,  the line  and the line  The function is undefined at since  hence the  area cannot be calculated | |
| Mathematical behaviours | Marks |
| * relates the integral to an area under the curve * states that  since is undefined at * concludes that the area cannot be calculated – it id not bounded | 1  1  1 |

**Question 19(b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines point of intersection of  and * states appropriate integral to determine first area * states appropriate integral to determine second area * evaluates one integral correctly * determines correct result to two decimal places | 1  1  1  1  1 |

**Question 19(c)**  **(3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence    Hence | |
| Mathematical behaviours | Marks |
| * determines average of two areas * states appropriate equation to be solved involving integral * solves equation and concludes solution | 1  1  1 |